

Comment on “No-core configuration-interaction model for the isospin- and angular-momentum-projected states” by Satuła, Bączyk, Dobaczewski, and Konieczka

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The authors state that results recently published by Carlsson and Toivanen “seem to contradict” the conclusions of some of our work. We show that this statement is based on a misinterpretation of the results of Carlsson and Toivanen.

In Ref. [1], Satuła, Bączyk, Dobaczewski, and Konieczka write the following. (We have changed the reference numbers in their text to correspond to the list of references at the end of this comment.)

“It is worth recalling here that in the context of searching for possible fingerprints of collective isoscalar pn -pairing phase in $N \approx Z$ nuclei, the isoscalar pairing, or deuteron-like correlations, were intensely discussed in the literature; see Refs. [2–4] and references cited therein. In particular, the isoscalar pn -pairing was considered to be the source of an additional binding energy that could offer a microscopic explanation of the so-called Wigner energy [5]—an extra binding energy along the $N = Z$ line, which is absent in the self-consistent MF mass models. In spite of numerous recent works following these early developments attempting to explain the isoscalar pn -pairing correlations and the Wigner energy (see Refs. [6–11] and references cited therein), the problem still lacks a satisfactory solution.

There are at least two major reasons for that: (i) an incompleteness of the HFB (HF) approaches used so far, which consider the pn mixing only in the particle-particle channel (see discussion in Ref. [12]), and (ii) difficulties in evaluating the role of beyond-mean-field correlations. Recently, within the RPA including pn correlations, the latter problem was addressed in Ref. [8]. Their systematic study of the isoscalar and isovector multiplets in magic and semi-magic nuclei rather clearly indicated a missing relatively strong $T = 0$ pairing. This seems to be in line with our NCCI model findings concerning description of $T = 0, I = 1$ states, but seems to contradict the conclusions of Refs. [7, 9].”

Our reading of Ref. [8] by Carlsson and Toivanen does not support that their findings “seem to contradict” our conclusions in Refs. [7, 9]. We model the masses of nuclei in the range of mass numbers $A = 24$ –100 with no or little neutron excess. In the doubly odd $N = Z$ nuclei we treat the lowest $T = 0$ and $T = 1$ states separately, where N , Z , and T denote the numbers of neutrons and protons and the isospin. To expose the variation of the observed and calculated masses with these variables we consider

four different combinations of the individual masses. In both works the model is isobarically invariant except for a phenomenological Coulomb contribution to the total mass.

In Ref. [7] the independent nucleon plus isovector pairing Hamiltonian is diagonalized exactly in a valence space formed by a small number of Nilsson levels. The deformation of each nucleus is taken from a previous calculation. A term proportional to $T(T + 1)$ is added to the calculated energies. It is concluded that this model reproduces the variation with A of the four mass combinations quite well except that the symmetry energy coefficient is overestimated when the coefficient of the $T(T + 1)$ term is fit to the mass difference of the lowest $T = 1$ and $T = 0$ states in the doubly odd $N = Z$ nuclei. A Hamiltonian with a certain additional isoscalar pairing interaction is also studied. A weak interaction of this type is found to have little effect on the results of calculations while with a larger coupling constant, the isoscalar pairing interaction destroys the reproduction of the $N = Z$ doubly odd doubly even mass differences.

In Ref. [9], to allow for larger valence spaces, the exact diagonalization of the isovector pairing Hamiltonian is replaced by the Hartree-Bogolyubov plus random phase approximation (RPA). The small valence spaces of Ref. [7] are used to verify the results based on this approach. Furthermore, a Strutinskij renormalization is applied. The model may thus be described from another point of view as a conventional Nilsson-Strutinskij calculation amended by an RPA correction based on the same pairing interaction as employed in the Bardeen-Cooper-Schrieffer term traditionally included in such calculations. Enlarging the valence space in this manner is found to eliminate the difficulty encountered in Ref. [7], specifically that is reproducing the mass difference of the lowest $T = 1$ and $T = 0$ states in the doubly odd $N = Z$ nuclei and the symmetry energy coefficient simultaneously. The 112 masses of doubly even nuclei in our sample are reproduced with a root mean square deviation of 0.95 MeV. Many features of the variation with A of the mass combinations are found explainable in terms of the shell struc-

ture, that is, the pattern of Nilsson levels. (Ref. [7] also has a discussion of the impact of shell structure on the so-called Wigner X . This is defined by a fit of the Coulomb reduced masses of nuclei with equal A and the lowest T by an expression proportional to $T(T+X)$ plus a constant.) The approximate $T(T+1)$ form of the binding energies is understood as a consequence of the spontaneous breaking of isospin symmetry by the mean field. The isospin symmetry breaking is caused to equal parts by the isovector pair field and the average potential, which is the source of the phenomenological $T(T+1)$ term of our theory.

We disagree with the statement that the findings of other authors “contradict” the results of our calculations. Further, we find no such statement in the article by Carlsson and Toivanen. In our reading, these authors do not address the Wigner energy at all; no mass is calculated. Their application of the RPA is to excitation energies, specifically the energies of neutron-proton pair and neutron hole-proton hole pair excitations of a doubly magic core. From the outset they abstain from reproducing the observed absolute values of these excitation energies, which would seem so constrain most directly the interaction strength. They consider the relative energies of the excited states with different angular momenta, which they fit by a fairly schematic two-component interaction. A ratio of about 1.4 of their isoscalar and isovector coupling constants is found to give the best fit in neighbors of doubly magic nuclei with $N \neq Z$. When the same interaction is applied to the neighbors of doubly magic nuclei with $N = Z$ their RPA calculations give imaginary excitation energies. The authors therefore dismiss these nuclei from their sample.

When an excitation energy calculated in the RPA is viewed in its dependence on the interaction strength, be-

fore becoming imaginary it must vanish. This means in the present context that the pair separation energy equals minus the sum of the neutron and proton chemical potentials of the core. The findings of Carlsson and Toivanen therefore imply that, in their model, the $N = Z$ doubly odd doubly even mass staggering vanishes in the neighborhoods of the doubly magic nuclei. This is certainly very different from what is observed. Thus, just contrary to contradicting our conclusions, the findings of Carlsson and Toivanen concur with some of our conclusions in Ref. [7]: A strong isoscalar pairing interaction induces a condensation of isoscalar pairs, which eliminates the $N = Z$ doubly odd doubly even mass staggering.

The authors of Ref. [1] cite Ref. [5] by Satuła, Dean, Gary, Mizutory, and Nazarewicz. They fail to mention our comment on this work in Ref. [9]. There we present the results of shell model calculation for $A = 48$ similar to those of Ref. [5] albeit restricted to the valence space including only the $1f_{7/2}$ shell. The valence space of Ref. [5] includes the shells $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$. Like the authors of Ref. [5] we find that the Wigner energy, defined as the deviation of the Coulomb reduced $T = 0$ mass from a quadratic fit to the $T = 2$ and $T = 4$ masses, decreases drastically when the interactions of isoscalar pairs are switched off. This turns out, however, to result from a decrease of the total symmetry energy. The Wigner X simultaneously *increases*. As ^{48}Cr is the central nucleus of the $1f_{7/2}$ shell, a similar analysis of the masses calculated by Satuła, Dean, Gary, Mizutory, and Nazarewicz in the larger valence space is not expected to lead to a qualitatively different conclusion. A definite answer to this question awaits the highly desirable publication of the individual calculated masses whence the published combinations were extracted.

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